The OGRE model

International Monetary Fund
Macro-structural training
Fiscal-structural clinic

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What is OGRE?

**Overlapping Generations and Retirement** = small open-economy dynamic general equilibrium model with demography and overlapping generations, unemployment, pay-as-you-go and fully funded pension plans, and informality in labor and product markets.

To investigate short- and long-run macroeconomic impacts of

1. demography
2. fiscal policies including pension reforms
3. neutralizing negative fiscal effects of aging by fiscal policies including pension reforms
4. and pension regime switches
No. 31.

- The structure of the model (closed economy)
- Channels based on a parametrized version:
  - macroeconomic impacts of aging (mortality, fertility)
  - fiscal multipliers
  - role of unemployment and informality
  - role of pension plans
- Technical Appendix
No. 32. and No. 2.

No. 32.:  
- Motivation:  
  - high-level of public debt and pension spending  
  - aging  
- We calibrate OGRE for Portugal and Spain (annual data) to study macroeconomic impacts of  
  - public policies under conditions of aging  
  - pension regime switches

No 2:  
- We extend OGRE by adding openness (small open-economy), and we explore the role of openness  
- We repeat the same exercise for Lithuania
References

- Baksa, D., and Z. Munkacsi, 2016, ”A Detailed Description of OGRE, the OLG Model”, BoL WP No. 31.

https://www.lb.lt/working_papers
https://www.lb.lt/occasional_papers
Follow-up works


- Baksa, D., C. Cuerpo Caballero, E. Martin Quilis, Z. Munkacsi, and A. Pastor Escribano, 2018, ”Pension reforms in Spain”.

Outline of this presentation

- Overview - done
- Data
- OGRE in nutshell
- Simulations
- Takeaway
- Appendix
Public debt

Source: Eurostat
Pension spending

Source: Eurostat

To compare: 4.5% on economic affairs and 7% on health
Aging

Source: Eurostat

Zsuzsa Munkacsi
Literature

- **Two types of OLG life-cycle models:**
  - classical:
    - finite number of periods, agents know when they die
    - Allais (1947), Diamond (1965), Samuelson (1958)
  - non-classical:
    - infinite number of periods
    - Blanchard (1985), Yaari (1965): agents die with a given probability in each period, but they do not know when
    - Weil (1989): a new cohort is born in each period
      + young and old agents
  - OGRE is a Gertler-type model
- **Not too many models with unemployment, but**
  - OGRE has unemployment (Blanchard and Gali, 2010)
  - besides Berger et al. (2009), OGRE is the only Gertler-type model with unemployment
OGRE is the first Gertler-type model with informality
  • the only classical model with informality is Keuschnigg et al. (2013)
There are papers on retirement age increase, but its impacts are not compared with those of other public policies (we do compare)
  • even though 28 out of 34 OECD countries planned or started to raise the retirement age (OECD, 2012)
Most models have either a PAYG plan, or study the impacts of a switch to FF
  • OGRE has both plans, and switches are also studied
There are several open-economy models in the literature
Defining the shadow economy

- In GDP: ”... all market-based legal production ... to avoid
  1. payment of ... taxes and
  2. social security contributions,
  3. having to meet ... minimum wages, maximum working hours, safety standards, etc., and
  4. complying with certain administrative obligations...”

- In employment: ”Shadow labor market includes all cases, where the employees or the employers, or both, occupy a shadow economy position.”

Source: Schneider (2012).
Size of shadow economy in GDP

Significant; larger in emerging and low-income than in developed countries.

Sources: Schneider (2005) and Schneider et al. (2010)
Demography

Population changes over time, in each period:

- new young people are born with a fertility rate
- the young retire with some probability
- the old pass away with a mortality rate

Cohort sizes: $20 \leq Y < \text{effective retirement age} \leq O$

The young:

- either work and pay labor income taxes or
- are unemployed and receive unemployment benefits

The old do not work, but receive public old-age pension benefits.
Both agents save and consume both formal and informal goods.
PAYG: pensions based on replacement rate and previous formal-sector labor income
Firms

- Cobb-Douglas production function with physical capital and labor
- Price setting with price rigidity costs (Rotemberg, 1982)
- To hire labor, an extra hiring cost is paid which induces unemployment
- Firms and workers bargain over wages
- Two sectors: formal (F) and informal (I)
Production

Formal
- Capital
- Labor
- Productivity

Informal
- Capital
- Labor
- Productivity

Expenditure
- Investment
- Consumption (formal and informal) / Savings

Capital producers

Young households
Formal vs. informal

- Williamson (1975): tax evasion + lower regulation

- Chart

- Lower regulation means:
  - higher hiring costs in formal sector
  - higher workers’ bargaining power over wages in formal sector
  - lower firing probability in formal sector
Formal

No tax evasion (VAT, labor income taxes)

Higher regulation

Higher productivity, public cons., int. trade

Informal

Tax evasion (VAT, labor income taxes)

Lower regulation

TRADE-OFFS

Government:
Direct impact on formal
No direct impact on informal

Interlinkages between formal and informal sectors

Shock absorber
Public sector

Government revenues:

- value-added taxes (on F goods)
- labor income taxes (in F sector)
  - employees: personal income tax, social security contributions
  - employers: social security contributions
- lump-sum taxes

Public expenditures:

- government consumption
- old-age pensions (in PAYG)
- unemployment benefits

Government deficit and debt.
Pension plans

Pay-as-you-go:

- initial pension based on a pension-wage replacement rate and previous years’ formal wage stream, then indexed by inflation
- public old-age pension expenditure financed by all public revenues

Fully funded:

- social security contributions of employees and employers collected in a pension fund → directly finance future public old-age pensions
New workers

Workers

Just-retired

Retired

FULLY FUNDED

PIT, VAT, and lump-sum to government (formal only)

SSC to pension fund (formal only)

Pensions based on previous savings and expected lifetime

PAYG

PIT, SSC, VAT, and lump-sum taxes to government (formal only)

Pensions based on replacement rate and previous formal-sector labor income

Back
Openness

- Small open-economy
- Young households can also save in foreign bonds
- Additional input in formal production are imports
- Re-exports
- Foreign interest rate risk premium (Schmitt-Grohe and Uribe, 2003)
Simulations

No. 31.:  

- Macroeconomic impacts of aging, i.e. higher old-age dependency ratio by reducing  
  - mortality rate  
  - fertility rate  

- Macroeconomic impacts of 1 per cent of SS/medium-term GDP size fiscal expansion by  
  - personal income taxes  
  - employees’ social security contributions  
  - employers’ social security contributions  
  - pension-wage replacement rate  
  - retirement age  
  - value-added taxes  
  - government consumption
Simulations cont.

No. 32. and No. 2.:

- Macroeconomic impacts of
  - neutralizing budgetary impacts of aging by fiscal policies
  - pension regime switches from PAYG to FF

In addition:

- Role of unemployment, informality and openness
- Dynare 4.4.3 and IRIS
- Long-run = 100 years, short-run = 10 years
Aging

Consumption (formal and informal) / Savings

New workers

PAYG: pensions based on replacement rate and previous formal-sector labor income

Workers

Just-retired

PAYG:
pensions based on replacement rate and previous formal-sector labor income

Consumption (formal and informal) / Savings

Retired

Consumption (formal and informal) / Savings

PIT, SSC, VAT, and lump-sum taxes to government (formal only)
# Impacts of aging - long run, parametrized

<table>
<thead>
<tr>
<th>Long-run effects of increasing old-age dependency ratio</th>
<th>GDP per capita (%)</th>
<th>Total household consumption per capita (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower mortality rate</td>
<td>-2.8</td>
<td>-3.8</td>
</tr>
<tr>
<td>Lower fertility rate</td>
<td>-2.6</td>
<td>1.2</td>
</tr>
<tr>
<td><strong>Young consumption-income ratio (pp)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lower mortality rate</td>
<td>-3.4</td>
<td>-6.6</td>
</tr>
<tr>
<td>Lower fertility rate</td>
<td>-5.1</td>
<td>9.1</td>
</tr>
<tr>
<td><strong>Old consumption-income ratio (pp)</strong></td>
<td></td>
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<tr>
<td>Lower mortality rate</td>
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<td>-3.8</td>
</tr>
<tr>
<td>Lower fertility rate</td>
<td>-4.1</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>Formal employment (%)</strong></td>
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<tr>
<td><strong>Informal employment (%)</strong></td>
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<td></td>
</tr>
<tr>
<td>Lower mortality rate</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Lower fertility rate</td>
<td>-1.4</td>
<td></td>
</tr>
</tbody>
</table>
Personal income tax increase

Production

Formal
- Capital (-)
+ Labor (-)
+ Productivity

Informal
+ Capital (+)
+ Labor (+)
+ Productivity

Expenditure

Investment

Consumption (relatively less formal)
Impacts of public policies - short run, parametrized, role of unemployment and informality
Pension regime switch - Portugal

<table>
<thead>
<tr>
<th>Long-run effects of a switch from PAYG to fully-funded in Portugal</th>
<th>Full reform</th>
<th>Partial reform</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP per capita (%)</td>
<td>1.5</td>
<td>0.8</td>
</tr>
<tr>
<td>Share of formal GDP in total GDP (%point)</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>Unemployment rate (%point)</td>
<td>-0.4</td>
<td>-0.2</td>
</tr>
<tr>
<td>Share of informal employment in total employment (%point)</td>
<td>-0.2</td>
<td>-0.1</td>
</tr>
<tr>
<td>Total household consumption per capita (%)</td>
<td>0.7</td>
<td>0.4</td>
</tr>
<tr>
<td>Share of young household consumption in total consumption (%point)</td>
<td>7.9</td>
<td>3.9</td>
</tr>
<tr>
<td>Gov. debt as a share of GDP (%point)</td>
<td>-0.6</td>
<td>-0.3</td>
</tr>
<tr>
<td>Implied replacement rate (%point)</td>
<td>-6.3</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

![Graph showing implied replacement rate (p.p.)](image-url)
Takeaway

Overlapping Generations and Retirement = small open-economy dynamic general equilibrium model with demography and overlapping generations, unemployment, pay-as-you-go and fully funded pension plans, and informality in labor and product markets

To investigate short- and long-run macroeconomic impacts of

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Demography

\[ N_t = N_t^Y + N_t^O \]
\[ N_t^Y = (1 - \omega_{t-1}^Y)N_{t-1}^Y + n_tN_{t-1}^Y \]
\[ N_t^O = (1 - \omega_{t-1}^O)N_{t-1}^O + \omega_{t-1}^Y N_{t-1}^Y \]
Retired cohort - optimization problem

\[ V^O(B_{a-1,t-1}(i)) = \]

\[ = \max \left\{ (1 + \epsilon_t^C) \left[ \frac{1}{1 - \gamma} \left\{ C_{a,t}^{O,F}(i) \right\}^{1-\gamma} + \frac{\chi}{1 - \gamma} \left\{ C_{a,t}^{O,I}(i) \right\}^{1-\gamma} \right] + \beta E_t (1 - \omega_t^O) V^O(B_{a,t}(i)) \right\} \]

subject to:

\[ (1 + \tau_t^C) C_{a,t}^{O,F}(i) + p_t^I C_{a,t}^{O,I}(i) + (1 - \omega_t^O) B_{a,t}(i) = \]

\[ = (1 + r_{t-1}) B_{a-1,t-1}(i) + TR_{a,t}^{PG,YO}(i) + TR_{a,t}^{FF,YO}(i) + \]

\[ + Profit_{a,t}(i) - T_{a,t}(i) \]
Retired cohort - optimization results

Euler equation:

\[ E_t C_{a+1,t+1}^O(i) = E_t C_{a,t}^O F(i)(1 + r_t) \left(1 + \frac{1}{\gamma} \Lambda_{t+1} \right) \]

\[ E_t \Lambda_{t+1} = E_t \left\{ \beta \frac{1 + \epsilon_{t+1}^C}{1 + \epsilon_t^C} \frac{1 + \tau_t^C}{1 + \tau_{t+1}^C} \right\}^{\frac{1}{\gamma}} \]

Informal-formal substitution:

\[ C_{a,t}^O i(i) = \gamma_t C_{a,t}^O F(i) \]

\[ \gamma_t = \left\{ \frac{1 + \tau_t^C}{\chi \frac{p_t^l}{p_t^l}} \right\}^{\frac{1}{\gamma}} \]

- positively affected by weight on informal goods in the utility function and value added tax rate
- and negatively affected by relative informal price
Retired cohort - individual consumption

\[ H^O_{t} C^O_{a,t}(i) = (TR_{PG, YO}^{P}(i) + TR_{FF, YO}^{F}(i))\Omega^O_t + \mathcal{I}^O_{a,t}(i) + (1 + r_{t-1})B^O_{a-1,t-1}(i) \]

\[ \mathcal{I}^O_{a,t}(i) = \text{Profit}^O_{a,t}(i) - T^O_{a,t}(i) + E_t \frac{1 - \omega^O_t}{1 + r_t} \mathcal{I}^O_{a+1,t+1}(i) \]

\[ H^O_t = (1 + \tau_t^C) + p_t^l \gamma_t + E_t (1 - \omega^O_t)(1 + r_t)^{1 - 1} \Lambda_{t+1} H^O_{t+1} \]

\[ \Omega^O_t = 1 + E_t \frac{1 - \omega^O_t}{1 + r_t} \Omega^O_{t+1} \]
Retired cohort - aggregate consumption

\[ H_t^O C_t^{O,F} = (TR_t^{PG} + TR_t^{FF}) \Omega_t^O + I_t^O + (1 + r_{t-1})(\omega_t^Y B_{t-1}^Y + B_{t-1}^O) + (1 + r_{t-1})q_t \omega_t^Y B_{t-1}^Y f_t^f + (1 + r_{t-1})q_t \omega_t^Y B_{t-1}^Y f_t^f \]

\[ C_t^{O,I} = \gamma_t C_t^{O,F} \]

\[ I_t^O = Profit_t^O - T_t^O + E_t \frac{1 - \omega_t^O}{(1 + r_t)(1 + g_{t+1}^N, O)} I_{t+1}^O \]

\[ Profit_t^O - T_t^O = (1 - \xi)(Profit_t - T_t) \]

- current old F consumption equals sum of discounted current and future pension benefits, other income, and current savings
- discounting also depends on the mortality rate \( \frac{1 - \omega_t^O}{1 + r_t} \)
- \( \omega_{t-1}^Y B_{t-1}^Y \) and \( \omega_{t-1}^Y B_{t-1}^Y f_t^f \) are included as \( \omega_{t-1}^Y \) share of the young got retired
Young cohort - optimization problem

\[ V_t^Y(B_{b-1,t-1}^Y(i), B_{b-1,t-1}^Y(i)) = \]
\[ = \max \left\{ (1 + \epsilon_t^C) \left[ \frac{1}{1 - \gamma} \left\{ C_{b,t}^{Y,F}(i) \right\}^{1-\gamma} + \frac{\chi}{1 - \gamma} \left\{ C_{b,t}^{Y,l}(i) \right\}^{1-\gamma} \right] + \right. \]
\[ + \beta E_t \left( (1 - \omega_t^Y) V_{t+1}^Y(B_{b,t}^Y(i), B_{b,t}^Y(i)) + \omega_t^Y V_{t+1}^O(B_{b,t}^O(i), B_{b,t}^O,f(i)) \right) \right\} \]

subject to:

\[ (1 + \tau_t^C) C_{b,t}^{Y,F}(i) + p_t^l C_{b,t}^{Y,l}(i) + (1 - \omega_t^Y) B_{b,t}(i) + q_t B_{b,t}^Y,f(i) + \]
\[ + \omega_t^Y (B_{b,t}^O(i) + q_t B_{b,t}^O,f(i)) = (1 + r_{t-1}) B_{b-1,t-1}(i) + (1 + r_{t-1}^f) q_t B_{b-1,t-1}^Y,f(i) + \]
\[ + (1 - \tau_t^{LW}) w_t^F L_{b,t}^F(i) + w_t^l L_{b,t}^l(i) + w_t^U U_{b,t}(i) + \text{Profit}_{b,t}(i) - T_{b,t}(i) \]
Young cohort - optimization results

Euler equations:

\[ E_t C_{b+1,t+1}(i) = E_t C_{b,t}^{Y,F}(i)(1 + r_t)^\frac{1}{\gamma} \Lambda_{t+1} \]
\[ E_t C_{0,t+1}(i) = E_t C_{b,t}^{Y,F}(i)(1 + r_t)^\frac{1}{\gamma} \Lambda_{t+1} \]

Informal-formal substitution:

\[ C_{b,t}^{Y,I}(i) = \Upsilon_t C_{b,t}^{Y,F}(i) \]

Uncovered interest rate parity:

\[ 1 + i_t = E_t(1 + i^{*}_t) \frac{S_{t+1}}{S_t} \]
Young cohort - individual consumption

\[ H_t^Y C_{b,t}^F(i) = I_{b,t}^Y(i) + \frac{I_{b,t}^YO(i)}{1 + r_t} + (1 + r_{t-1})B_{b-1,t-1}^Y(i) + (1 + r_{t-1}^f)q_t B_{b-1,t-1}^Y(i) \]

\[ I_{b,t}^Y(i) = Inc_{b,t}(i) + E_t \frac{1 - \omega_t^Y}{1 + r_t} I_{b,t+1}^Y(i) \]

\[ Inc_{b,t}(i) = (1 - \tau_t^{LW}) w_t^F L_{b,t}^F(i) + w_t^L L_{b,t}^L(i) + w_t^U U_{b,t}(i) + Profit_{b,t}^Y(i) - T_{b,t}^Y(i) \]

\[ I_{b,t}^YO(i) = E_t \omega_t^Y \left( (TR_{0,t+1}^{PG,YO}(i) + TR_{0,t+1}^{FF,YO}(i)) \Omega_{t+1}^O + I_{0,t+1}^O(i) \right) + \]
\[ + \frac{1 - \omega_t^Y}{1 + r_{t+1}} I_{b,t+1}^YO(i) \]

\[ H_t^Y = (1 + \tau_t^C) + p_t^I \gamma_t + \]
\[ + E_t (1 + r_t) \frac{1}{\gamma - 1} \Lambda_{t+1} \left( (1 - \omega_t^Y) H_{t+1}^Y + \omega_t^Y H_{t+1}^O \right) \]
Young cohort - aggr. cons. and budget constraint

\[ H_t^{Y} C_t^{Y,f} = I_t^{Y} + \frac{I_t^{YO}}{1 + r_t} + (1 + r_{t-1})(1 - \omega_t^{Y})B_{t-1}^Y + \]
\[ + (1 + r_{t-1}^f)(1 - \omega_t^{Y})q_t B_{t-1}^{Y,f} \]
\[ C_t^{Y,l} = C_t^{Y,f} \gamma_t \]

\[ I_t^{Y} = Inc_t + E_t \frac{1 - \omega_t^{Y}}{(1 + r_t)(1 + g_{t+1}^{N,Y})}I_{t+1}^{Y} \]

\[ Inc_t = (1 - \tau_t^{LW})w_t^{F}L_t^{F} + w_t^{I}L_t^{I} + w_t^{U}U_t + Profit_t^{Y} - T_t^{Y} \]
\[ Profit_t^{Y} - T_t^{Y} = \xi(Profit_t - T_t) \]
Young cohort - aggr. cons. and budget constraint cont.

\[ I_t^{YO} = E_t \left( (TR_{t+1}^{PG,YO} + TR_{t+1}^{FF,YO})\Omega_{t+1} + \frac{\omega^Y_t}{(1 + g_{t+1}^{N,Y}) s_{t+1}} I_{t+1}^O \right) + \]

\[ + E_t \frac{1 - \omega^Y_t}{(1 + r_{t+1})(1 + g_{t+1}^{N,Y})} I_{t+1}^{YO} \]

\[ (1 + \tau^C_t) C_t^{Y,F} + p^l_t C_t^{Y,I} + B_t^Y + q_t B_{t,f}^Y = \]

\[ = Inc_t + (1 + r_{t-1})(1 - \omega^Y_{t-1}) B_{t-1}^Y + (1 + r_{t-1}^f)(1 - \omega^Y_{t-1}) q_t B_{t-1}^Y, f \]

- current F consumption of the young equals sum of discounted stream of current and future income, and current savings
- discounting depends on retirement and mortality rates
Good producers (formal) - optimization problem

\[ V(P_{t-1}^F(j), L_{t-1}^F(j)) = \max \left\{ \text{profit}_t^F(j) + E_t \frac{V(P_t^F(j), L_t^F(j))}{1 + i_t} \right\} \]

subject to:

\[
\begin{align*}
\text{profit}_t^F(j) & = P_t^F(j) Y_t^F(j) - R_t^K^F K_{t-1}^F(j) - (1 + \tau_t^{SSCF}) W_t^F L_t^F(j) - \\
& \quad - HC_t^F H_t^F(j) - P_t^F(j) Y_t^F R \left( \frac{P_t^F(j)}{P_{t-1}^F(j)} \right) - S_t P_t^M,^* M_t^Y(j)
\end{align*}
\]

\[
Z_t(j) = K_{t-1}^F(j) \alpha_t^F (A_t^F L_t^F(j))^{(1-\alpha_t^F)}
\]

\[
Y_t^F(j) = \left( \Lambda^{\frac{1}{\nu}} Z_t(j) \frac{\nu - 1}{\nu} + (1 - \Lambda) \frac{1}{\nu} M_t^Y(j) \frac{\nu - 1}{\nu} \right)^{\frac{\nu}{\nu-1}}
\]

\[
Y_t^F(j) = \left( \frac{P_t^F(j)}{P_t^F} \right)^{-\varphi} Y_t^F
\]
Good producers (formal) - optimization problem cont.

subject to cont.:

\[ L_t^F(j) = (1 - \text{pr}_t^{F,F})L_{t-1}^F(j) + H_t^F(j) \]

\[
R \left( \frac{P_t^F(j)}{P_{t-1}^F(j)} \right) = \frac{\phi P}{2} \left( \frac{P_t^F(j)}{P_{t-1}^F(j)} \right)^2
\]

\[ = \frac{\phi P}{2} \left( \frac{1 + \pi_t^F}{(1 + \pi_{t-1}^F)^\gamma} - 1 \right)^2 \]

\[ H:C_t^F = \kappa_t^F \left( \text{pr}_t^{H,F} \right)^{\alpha_{HC}} \]

\[ \text{pr}_t^{H,F} = \frac{H_t^F}{U_{t-1} + \text{pr}_t^{F,F}L_{t-1}^F + \text{pr}_t^{F,I}L_{t-1}^I} \]
Good producers (formal) - optimization results

Capital and labor demand, marginal cost and import demand:

\[ K_{t-1}^F = \alpha^F \frac{mc_t^Z}{r^F_{K,F}} Z_t \]

\[ L_t^F = (1 - \alpha^F) \frac{mc_t^Z}{\tilde{w}_t} Z_t \]

\[ mc_t^Z = \left( \frac{r^F_{K,F}}{\alpha^F} \right)^{\alpha^F} \left( \frac{\tilde{w}_t}{A_t^F (1 - \alpha^F)} \right)^{(1 - \alpha^F)} \]

\[ M_t^Y = (1 - \Lambda) \left( \frac{mc_t^F}{q_t} \right)^\nu Y_t^F \]
Good producers (formal) - optimization results cont.

where

\[ \tilde{w}_t = (1 + \tau_t^{SSC})w_t^F + hc_t^F - E_t \frac{hc_{t+1}^F(1 - pr_{t+1}^F,F)}{1 + r_t} \]

\[ mc_t^F = \left( \Lambda mc_t^Z 1^{-\nu} + (1 - \Lambda)q_t 1^{-\nu} \right) \frac{1}{1^{-\nu}} \]

Pricing decision:

\[ 1 + \frac{1}{\varphi - 1} \left\{ R \left( \frac{1 + \pi_t^F}{(1 + \pi_{t-1}^F)\gamma} \right) + R' \left( \frac{1 + \pi_t^F}{(1 + \pi_{t-1}^F)\gamma} \right) \frac{1 + \pi_t^F}{(1 + \pi_{t-1}^F)\gamma} \right\} - \]

\[ -E_t \frac{1}{\varphi - 1} \frac{Y_{t+1}^F}{Y_t^F} \left( \frac{1 + \pi_{t+1}^F}{(1 + \pi_t^F)\gamma} \right) \left( \frac{1 + \pi_{t+1}^F}{(1 + \pi_t^F)\gamma} \right) \frac{1 + \pi_t^F}{1 + r_t} = \frac{\varphi}{\varphi - 1} mc_t^F \]
Bargaining - value functions

Worker value functions:

\[ V_t^F = (1 - \tau_t^{LW})w_t^F + E_t \frac{1}{1+r_t} \left[ (1 - pr_{t+1}^{F,F} + pr_{t+1}^{F,F} pr_{t+1}^{H,F}) V_{t+1}^F + 
  + pr_{t+1}^{F,F} pr_{t+1}^{H,I} V_{t+1}^I + pr_{t+1}^{F,F} (1 - pr_{t+1}^{H,F} - pr_{t+1}^{H,I}) V_{t+1}^U \right] \]

\[ V_t^I = w_t^I + E_t \frac{1}{1+r_t} \left[ (1 - pr_{t+1}^{F,I} + pr_{t+1}^{F,I} pr_{t+1}^{H,I}) V_{t+1}^I + 
  + pr_{t+1}^{F,I} pr_{t+1}^{H,F} V_{t+1}^F + pr_{t+1}^{F,I} (1 - pr_{t+1}^{H,F} - pr_{t+1}^{H,I}) V_{t+1}^U \right] \]

\[ V_t^U = w_t^U + E_t \frac{1}{1+r_t} \left[ (1 - pr_{t+1}^{H,I} - pr_{t+1}^{H,F}) V_{t+1}^U + 
  + pr_{t+1}^{H,F} V_{t+1}^F + pr_{t+1}^{H,I} V_{t+1}^I \right] \]

Firm value functions = hiring costs.
Bargaining (formal) - optimization problem and result

Problem:

\[
\max_{w_t^F} (V_t^F - V_t^U)^{\sigma^F} h c_t^{F^{1-\sigma^F}}
\]

Result:

\[
\frac{\sigma^F}{1 - \sigma^F} h c_t^F \frac{1 - \tau_{t}^{LW}}{1 + \tau_{t}^{SSCF}} = (1 - \tau_{t}^{LW}) w_t^F - w_t^U + \\
E_t \frac{1}{1 + r_t} \left[ (1 - pr_{t+1}^{F,F})(1 - pr_{t+1}^{H,F}) \left( \frac{\sigma^F}{1 - \sigma^F} h c_{t+1}^F \frac{1 - \tau_{t+1}^{LW}}{1 + \tau_{t+1}^{SSCF}} \right) - \\
-(1 - pr_{t+1}^{F,F})pr_{t+1}^{H,I} \left( \frac{\sigma^I}{1 - \sigma^I} h c_{t+1}^I \right) \right]
\]
Public sector with PAYG

\[ Rev_t = \tau_t^C C_t^F + \tau_t^L w_t^F L_t^F + T_t \]
\[ \tau_t^L = \tau_t^{PL} + (1 - \Xi)(\tau_t^{SSCW} + \tau_t^{SSCF}) \]
\[ \tau_t^{LW} = \tau_t^{PL} + \tau_t^{SSCW} \]
\[ Exp_t = Gov_t + w_t^U U_t + TR_{t}^{PG} \]
\[ B_t + Rev_t = (1 + r_{t-1})B_{t-1} + Exp_t \]
\[ PB_t = Rev_t - Exp_t \]
\[ GB_t = PB_t - r_{t-1}B_{t-1} \]
\[ T_t = \eta T + (1 - \eta) \left[ \rho T_{t-1} + (1 - \rho)(GB_t^{Target} - GB_t) \right] \]
\[ IB_t^Y = \frac{1}{Y} w_{t-1}^F L_{t-1}^F + \frac{Y - 1}{Y} (1 - \omega_{t-2})IB_{t-1}^Y \]
\[ TR_{t}^{PG,YO} = \nu_t \omega_{t-1}^Y IB_{t-1}^Y \]
\[ TR_t^{PG} = TR_{t}^{PG,YO} + (1 - \omega_{t-1}^O)TR_{t-1}^{PG} \]
FF pension system

\[\text{Exp}_t = \text{Gov}_t + w_t^U U_t\]
\[B_{t-1}^{Y,*} = \Xi (\tau_t^{SSCW} + \tau_t^{SSCF}) w_t^F L_t^F + (1 + r_{t-1})(1 - \omega_t^{Y}) B_{t-1}^{Y,*}\]
\[(1 + r_{t-1})\omega_{t-1} B_{t-1}^{Y,*} = TR_t^{FF,YO} \Omega_t^O\]
\[TR_t^{FF} = TR_t^{FF,YO} + (1 - \omega_t^{O}) TR_{t-1}^{FF}\]
\[(1 + r_{t-1})\omega_{t-1} B_{t-1}^{Y,*} + (1 + r_{t-1}) B_{t-1}^{O,*} = TR_t^{FF} + B_t^{O,*}\]
\[B_t^* = B_t^{Y,*} + B_t^{O,*}\]
Small open-economy

Exports:

\[ X_t = Y_t^* \left( \frac{p^X_t}{q_t} \right)^{-\theta} \]

\[ X_t = \left[ \zeta Y_t^F \eta^X + (1 - \zeta) M_t^X \eta \right]^\frac{1}{\eta} \]

Risk premium:

\[ 1 + r_t^f = (1 + r_t^*) \exp(\chi^{Debt} \left( \frac{B_{t-1}^{Y,f}}{GDP_{t-1}} - \frac{B^{Y,f}}{GDP} \right)) \exp(\epsilon_{t}^{r^*}) \]

Trade balance and foreign debt:

\[ TB_t = p^X_t X_t - q_t M_t \]

\[ \frac{TB_t}{q_t} = B_{t}^{Y,f} - B_{t-1}^{Y,f}(1 + r_{t-1}^f) \]
Impacts of mortality drop - short run, parametrized
Impacts of fertility drop - short run, parametrized
Impacts of public policies - long run, parametrized

### Long-run effects of fiscal expansion

<table>
<thead>
<tr>
<th>Fiscal expansion with</th>
<th>GDP per capita (%)</th>
<th>Unemployment rate (%point)</th>
<th>Total household consumption per capita (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PAYG</td>
<td>FF</td>
<td>PAYG</td>
</tr>
<tr>
<td>Personal income tax</td>
<td>1.2</td>
<td>1.2</td>
<td>-0.4</td>
</tr>
<tr>
<td>Employee SSC</td>
<td>1.2</td>
<td>1.0</td>
<td>-0.4</td>
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<tr>
<td>Employer SSC</td>
<td>0.9</td>
<td>0.7</td>
<td>-0.3</td>
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<tr>
<td>Pension-wage replacement rate</td>
<td>0.0</td>
<td>-</td>
<td>0.0</td>
</tr>
<tr>
<td>Retirement probability</td>
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<td>-0.4</td>
<td>0.0</td>
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<tr>
<td>Value added tax</td>
<td>0.0</td>
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<tr>
<td>Government consumption exp./GDP</td>
<td>0.1</td>
<td>0.1</td>
<td>0.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fiscal expansion with</th>
<th>Share of young household consumption in total consumption (%point)</th>
<th>Share of formal GDP in total GDP (%point)</th>
<th>Share of informal employment in total employment (%point)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Retirement probability</td>
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<td>-0.1</td>
<td>0.0</td>
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<tr>
<td>Value added tax</td>
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<td>0.4</td>
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<tr>
<td>Government consumption exp./GDP</td>
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<td>-0.1</td>
<td>0.5</td>
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Impacts of public policies - short run, parametrized
Impacts of public policies - short run, parametrized cont.
Impacts of public policies - short run, parametrized, PAYG vs. fully funded
## Impacts of public policies - long run, parametrized, role of unemployment and informality

### Long-run effects of fiscal expansion in a PAYG regime

<table>
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<td></td>
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<th>Fiscal expansion with</th>
<th>Total household consumption per capita (%)</th>
<th>Share of young household consumption in total consumption (%point)</th>
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<td></td>
<td>Baseline</td>
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<tr>
<td>Personal income tax</td>
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<td>Value added tax</td>
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</tr>
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<td>Government consumption exp./GDP</td>
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<td>-1.9</td>
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</table>
### Impacts of public publicies - long run, Portugal

#### Long-run effects of a 5 pp increase in the old-age dependency ratio in a PAYG regime in Portugal

<table>
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<tr>
<th>If the government consolidates with</th>
<th>GDP per capita (%)</th>
<th>Share of formal GDP in total GDP (%point)</th>
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<tbody>
<tr>
<td>No consolidation</td>
<td>-3.6</td>
<td>0.4</td>
</tr>
<tr>
<td>Personal income tax and employee SSC</td>
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</tr>
<tr>
<td>Employer SSC</td>
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<td>0.6</td>
</tr>
<tr>
<td>Pension-wage replacement rate</td>
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<tr>
<td>Retirement probability</td>
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<tr>
<td>Value added tax</td>
<td>-3.6</td>
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</tr>
<tr>
<td>Government consumption exp./GDP</td>
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<td>Fertility rate</td>
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Impacts of public publicies - short run, Portugal